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Electromagnetic Wave Scattering by Fractal Surface

A. O. Pinchuk and V. Gozhenko

Institute of Surface Chemistry
National Academy of Science of Ukraine
Kyiv, Nauki av., 31, Ukraine, 03022
Fax: (380) 44 264 04 46, e-mail: onmf@serv.biph.kiev.ua

Abstract

We have considered some peculiarities of light reflection by several types of fractal surfaces. The two-dimensional Weierstrass function for modeling of fractal surfaces was used. The computer simulation of real rough surfaces was performed by means of this function. The electromagnetic waves scattering indicatrix was obtained for the concrete fractal surfaces based on the scalar Kirchhoff theory. The computer simulation of light reflection by fractal surfaces was performed.

1. Introduction

All the surfaces are rough in some way or another. Therefore the studying of the scattering of electromagnetic waves by rough surfaces is an important and interesting theoretical and experimental task. The investigation of the peculiarities of such scattering is an important first of all for a non-destructive control of real surfaces. Real surfaces are most adequately described by fractal functions as they are neither pure deterministic nor pure casual. Last time the fractal nature of a number of different surfaces have been experimentally determined (sea surface and sea bottom, Earth relief, cloud surfaces, thin film surfaces, deposited on the substrate, etc.).

The aim of this work was the simulation of the electromagnetic waves scattering indicatrix based on the scalar Kirchhoff theory for particular surfaces. The analogues calculations were performed by others authors [2], but our results have some distinctive features, particularly in the expression for the averaged scattering coefficient there are some additional terms which can significantly influence on the resulting scattering indicatrix at a certain geometry of the experiment.

2. Theory

We have chosen the two-dimensional Weierstrass function $z(x,y)$ for the simulation of rough surfaces

$$z(x,y)=c_w\sum_{n=0}^{N-1}\sum_{m=1}^{M}q^{(D-3)n}\sin\left\{Kq^n\left[x\cos\frac{2\pi m}{M}+y\sin\frac{2\pi m}{m}\right]+\phi_{nm}\right\}, \tag{1}$$

were c_w is a normalizing factor; $q>1$ is the fundamental spatial frequency; K is the fundamental wavenumber of the surface; D is the fractal dimension ($2<D<3$); N,M are numbers of tones; ϕ_{nm}

is a phase term. The example of a rough surface simulated using the function (1) is shown on Fig. 1(a).

Let us consider the wave falling on the rough surface S at an angle Θ_1 and scattering at a polar angle Θ_2 and at an azimuth angle Θ_3. We will be finding the scattered field $E_s(\vec{r},t)$ based on the scalar Kirchhoff method [4]:

$$E_s(\vec{r}) = -ikrF(\Theta_1,\Theta_2\Theta_3)\frac{\exp(ikr)}{2\pi r}\int_{S_0}\exp[ik\phi(x_0,y_0)]dx_0dy_0 + E_e(\vec{r}), \tag{2}$$

where k is the wavenumber of the incident wave,

$$F(\Theta_1,\Theta_2,\Theta_3) = -\frac{R}{2C}(A^2+B^2+C^2)$$

is the angle factor, R is the reflection coefficient,

$$A = \sin\Theta_1 - \sin\Theta_2\cos\Theta_3,$$

$$B = -\sin\Theta_2\sin\Theta_3,$$

$$C = -\cos\Theta_1 - \cos\Theta_2,$$

$$\phi(x_0,y_0) = Ax_0 + By_0 + Ch(x_0,y_0)$$

is the phase function,

$$h(x_0,y_0) = z(x_0,y_0),$$

$$E_e(\vec{r}) = -\frac{R}{C}\cdot\frac{\exp(ikr)}{4\pi r}(AI_1+BI_2)$$

is the bound term,

$$I_1 = \int_{-Y}^{Y}\left[e^{ik\phi(X,y_0)} - e^{ik\phi(-X,y_0)}\right]dy_0, \qquad I_2 = \int_{-X}^{X}\left[e^{ik\phi(x_0,Y)} - e^{ik\phi(x_0,-Y)}\right]dx_0. \tag{3}$$

The above formalism is valid under the following conditions[4]: the incident wave is monochromatic and plane; the scattering surface is rough inside a certain square and smooth outside its; the surface dimensions are much large than the incident wavelength; all the surface points have finite gradient; the reflection coefficient is a constant across the surface area; the scattered field is observed far from the surface.

After some transformations from Eq.(2) considering Eq.(1) we obtain the expression for the average scattering coefficient

$$\langle\rho_s\rangle \equiv \frac{\langle I_s\rangle}{I_0} \tag{4}$$

where $\langle I_s\rangle = \langle \vec{E}_s\vec{E}_s^*\rangle$, $I_0 = \left(\frac{2kXY\cos\Theta_1}{\pi r}\right)^2$ is the intensity of a wave reflected from the respective smooth surface. Neglecting the terms higher than ξ_u^2 ($\xi_u \equiv kc_w Cq^{(D-3)u}$), this expression has the following approximate form

$$\langle \rho_s\rangle \approx \left[\frac{F(\Theta_1,\Theta_2,\Theta_3)}{\cos\Theta_1}\right]^2 \{ \left[1-(k\sigma C)^2\right]\mathrm{sinc}^2(kAX)\,\mathrm{sinc}^2(kBY)+$$

$$+\frac{1}{2}c_f^2\sum_{nm} q^{2(D-3)n}\,\mathrm{sinc}^2\left[\left(kA+Kq^n\cos\frac{2\pi m}{M}\right)X\right]\mathrm{sinc}^2\left[\left(kB+Kq^n\sin\frac{2\pi m}{M}\right)Y\right]\}+$$

$$+\left[\frac{R}{2C\cos\Theta_1}\left(A^2+B^2\right)\right]^2\mathrm{sinc}^2(kAX)\,\mathrm{sinc}^2(kBY), \tag{5}$$

where, σ is the root mean square height of the surface roughness, X and Y are the dimensions reflecting area

$$c_f \equiv kc_w C = k\sigma C\left[\frac{2}{M}\cdot\frac{1-q^{2(D-3)}}{1-q^{2N(D-3)}}\right]^{\frac{1}{2}},$$

$$\mathrm{sinc}\,x \equiv \frac{\sin x}{x};\ \sum_{nm} \equiv \sum_{n=1}^{N-1}\sum_{m=0}^{M}.$$

3. Numerical results

We have calculated the averege reflected coefficient $\langle\rho_s\rangle$ as a function of Θ_2 and Θ_3 (the scattering indicatrix) based on Eq. (5). We have assumed that $R=1$, in other words we have not considered the real dependence of the reflectance R on the wavelength λ and on the incident angle Θ_1. The example of the scattering indicatrix is shown on Fig. 1 (b).

4. Conclusion

The analysis of the obtained results leads to some inferences:

- The waves scattering is symmetrical relatively the plane of incidence;
- The most intensity of the scattered waves is observed in the specular direction;
- There are others directions, where some splashes of intensity are observed;
- The picture of the reflection complicates with increasing of the surface large scale homogeneity.

These peculiarities are due to combination of chaotic character and self-similarity of the real surface relief.

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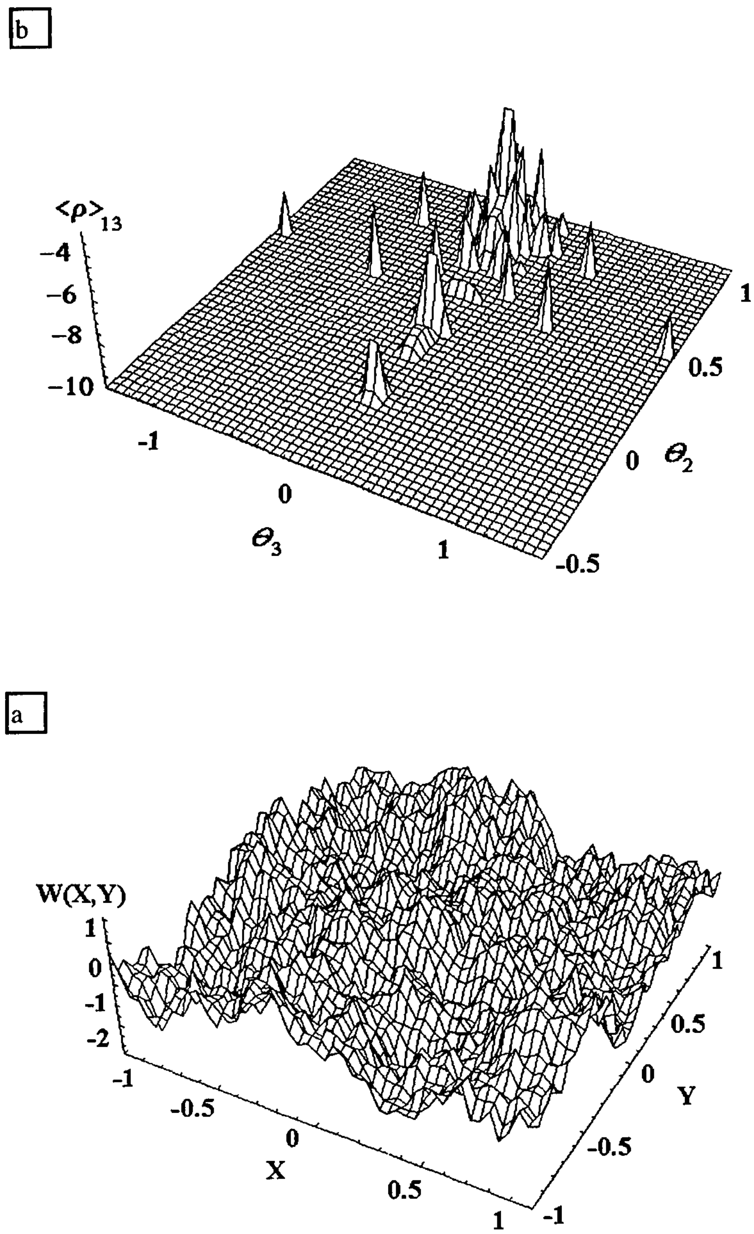

Fig. 1 **a)** The simulation of the fractal surface by means of Weierstrass function $K = 6.3$, $N = M = 5$, $D = 2.5$; **b)** The reflection coefficient $\log\langle\rho_s\rangle$ for the fractal surface with $D = 2.5$, $q = 1.8$; $N = M = 10$.

References

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